

A New Approach to Superstring Field Theory

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Abstract: I review the construction of an action for open superstring field theory which does not suffer from the contact term problems of other approaches. I also discuss a possible generalization of this action for closed superstring field theory. (Talk presented at 32nd Ahrenshoop International Symposium on Elementary Particle Theory in Buckow, Germany)

1 Problems with Conventional Approach

The construction of a field theory action for the superstring is an important problem since it may lead to information about non-perturbative superstring theory which is unobtainable from the on-shell perturbative S-matrix. This information might be useful for understanding the non-perturbative dualities of the superstring. Although there was much activity ten years ago concerning a field theory action for the bosonic string, there was not much progress on constructing a field theory action for the superstring. For reasons described below, there was no obvious generalization of the bosonic string field theory action to the superstring. Since the non-perturbative dualities of the superstring are not expected to have any analog for the bosonic string, it is not too surprising that not much was learned about these dualities by studying bosonic string field theory.

The covariant string field theory action for the bosonic string is based on a BRST operator Q and a string field A . In Witten's approach to open string field theory, the gauge-invariant action is [1]

$$S_{open} = \int [AQA + A^3] \quad (1)$$

where string fields are glued together at their midpoint. This gluing prescription preserves the length of the open string, so one can fix the open string length in this approach. Witten's gluing procedure can be generalized to closed strings by requiring that two closed strings only interact when half of one closed string overlaps with half of the other closed string, so that the length of the resulting closed string is the same as the length of each initial closed string. Using this gluing procedure, gauge invariance implies that the field theory action is non-polynomial in A , i.e. [4]

$$S_{closed} = \int [A(c_0 - \bar{c}_0)(Q + \bar{Q})A + A^3 + \dots] \quad (2)$$

where ... includes contributions from all orders in A . The factor $(c_0 - \bar{c}_0)$ is necessary in the kinetic term from ghost-number counting since A carries ghost-number 2 and the closed string ghost-number anomaly is +6. Because of the $(c_0 - \bar{c}_0)$ factor, this closed string action is gauge invariant under $\delta A = (Q + \bar{Q})\Lambda$ only if both A and Λ satisfy the constraints $(b_0 - \bar{b}_0)A = (b_0 - \bar{b}_0)\Lambda = 0$.

There is another covariant approach to bosonic string field theory in which open strings are glued at their endpoints and closed strings are glued at one point only [2][3]. This approach is similar to light-cone string field theory where the length of the string is the momentum in the light-cone direction. In this covariantized light-cone approach, the length of the string becomes a free parameter which is conserved in interactions, i.e. the length of the final string is equal to the sum of the lengths of the two initial strings. Although this free parameter can be gauged away on-shell, it is unclear how to treat the infinities caused by integration over this parameter in the functional integral. Nevertheless, it is interesting to note that using this gluing prescription, it is possible to construct a gauge-invariant cubic closed string field theory action of the form

$$S_{closed} = \int [A(c_0 - \bar{c}_0)(Q + \bar{Q})A + A^3]. \quad (3)$$

As before, $(b_0 - \bar{b}_0)A = 0$ is required for gauge invariance. One can also construct a gauge-invariant open string field theory action using the end-point gluing prescription, however, unlike the action of (1) (but like the light-cone open string action), it is not cubic.

In generalizing these approaches to superstring field theory, the main difficulty comes from the requirement that the string field carries a definite "picture". Recall that each physical state of the superstring is represented

by an infinite number of BRST-invariant vertex operators in the covariant RNS formalism [5]. To remove this infinite degeneracy, one needs to require that the vertex operator carries a definite picture, identifying which modes of the (β, γ) ghosts annihilate the vertex operator. For open superstring fields, the most common choice is that all Neveu-Schwarz string fields carry picture -1 and all Ramond string fields carry picture $-\frac{1}{2}$. For closed superstring fields, one has an analogous choice of (left,right) picture corresponding to the choice of Neveu-Schwarz or Ramond in the (left,right) sectors.

Since the total picture must equal -2 for open superstrings, the obvious generalization of the action (1) is [6]

$$S = \int [A_{NS} Q A_{NS} + A_R Q Y A_R + Z A_{NS}^3 + A_{NS} A_R A_R] \quad (4)$$

where $Z = \{Q, \xi\}$ is the picture-raising operator of picture $+1$ and $Y = c\partial\xi e^{-2\phi}$ is the picture-lowering operator of picture -1 . However, as shown by Wendt [7], the action of (4) is not gauge-invariant because of the contact-term divergences occurring when two Z 's collide. Although one can choose other pictures for the string field A which change the relative factors of Z and Y [8], there is no choice for which the action is cubic and gauge-invariant [9] [10]. One way to make the action gauge-invariant would be to introduce an infinite number of contact terms to cancel the divergences coming from colliding Z 's. However, the coefficients of these contact terms would have to be infinite in the classical action since the divergences are present already in tree-level amplitudes. Note that an infinite number of divergent contact terms are also expected in light-cone superstring field theory (either in the RNS or Green-Schwarz formalisms) to cancel the divergences when interaction points collide [11].

For the closed superstring, the total left and right-moving pictures must both equal -2 , so the generalization of the actions of (2) and (3) is [12]

$$\begin{aligned} S_{closed} = \int [& A_{NS,NS}(c_0 - \bar{c}_0)(Q + \bar{Q})A_{NS,NS} + \\ & A_{NS,R}(c_0 - \bar{c}_0)(Q + \bar{Q})\bar{Y}A_{NS,R} + A_{R,NS}(c_0 - \bar{c}_0)(Q + \bar{Q})YA_{NS,R} + \\ & A_{R,R}(c_0 - \bar{c}_0)(Q + \bar{Q})Y\bar{Y}A_{R,R} + Z\bar{Z}A_{NS,NS}^3 + \dots]. \end{aligned} \quad (5)$$

Since (5) contains Z 's in the interactions, this action naively suffers from the same contact-term divergences as the action of (4). However, using the closed-string gluing prescription of (2), interaction points never collide

so there is no problem. But these contact-term divergences are a problem using the covariantized light-cone gluing prescription of (3) which allows colliding interaction points (as in light-cone string field theory).

A problem with (5) which exists using either gluing prescription is that the picture-lowering operators Y and \bar{Y} do not commute with $(b_0 - \bar{b}_0)$. This implies that the Ramond contribution to the action is not gauge-invariant under $\delta A = (Q + \bar{Q})\Lambda$ even at the quadratic level. Such a problem with the Ramond sector is not surprising since the Type IIB Ramond-Ramond sector contains a massless chiral four-form state for which it is extremely difficult to construct a kinetic action.

2 New Approach for the Open Superstring

In these proceedings, a new approach to superstring field theory will be proposed which uses two BRST-like operators instead of just one. This approach is based on the fact that any critical N=1 superconformal field theory (such as the ten-dimensional superstring) can be described by a critical N=2 superconformal field theory. This N=1 \rightarrow N=2 embedding was described in reference [13] where it was shown that any physical N=1 vertex operator can be represented by a physical N=2 vertex operator, and the scattering amplitudes coincide using either the N=1 or N=2 prescriptions for computation. Furthermore, it was shown in reference [14] that, after twisting, N=2 physical vertex operators and scattering amplitudes can be computed without introducing N=2 ghosts.

This ghost-free prescription was called an N=4 topological prescription since it uses the (small) N=4 superconformal generators which can be constructed from any set of critical N=2 superconformal generators. The four fermionic N=4 generators will be labeled as $(G^+, G^-, \tilde{G}^+, \tilde{G}^-)$ where G^+ and G^- are the original fermionic N=2 generators, $\tilde{G}^+ = [e^{\int J}, G^-]$, $\tilde{G}^- = [e^{-\int J}, G^+]$, and $(e^{\int J}, J, e^{-\int J})$ are the SU(2) generators constructed from the original U(1) generator J . After twisting, (G^+, \tilde{G}^+) carry conformal weight +1 and (G^-, \tilde{G}^-) carry conformal weight +2.

There are three critical N=2 superconformal field theories which will be relevant here. The first is the self-dual string which describes self-dual Yang-Mills (open) or self-dual gravity (closed) in $D = (2, 2)$ [15]. The left-moving worldsheet fields of the self-dual string consist of $(x^{\pm a}, \psi^{\pm a})$ where $a = 1$ to 2. For the self-dual string, the left-moving N=4 fermionic

generators are

$$G^+ = \partial x^{-a} \psi^{+a}, \quad \tilde{G}^+ = \partial x^{+a} \psi^{+a}, \quad G^- = \partial x^{+a} \psi^{-a}, \quad \tilde{G}^- = \partial x^{-a} \psi^{-a}. \quad (6)$$

A second critical N=2 superconformal field theory is given by the N=2 embedding of the RNS superstring. The worldsheet fields are the usual RNS worldsheet variables and the N=4 fermionic generators are [16][14]

$$G^+ = j_{BRST}, \quad \tilde{G}^+ = \eta, \quad G^- = b, \quad \tilde{G}^- = \{Q, b\xi\} = bZ + \xi L \quad (7)$$

where $Q = \int j_{BRST}$ is the standard BRST charge of the N=1 superstring, η and ξ come from bosonizing the (β, γ) ghosts as $(\beta = \partial\xi e^{-\phi}, \gamma = \eta e^{\phi})$ [5], Z is the picture-raising operator, and L is the RNS stress-tensor.

A third critical N=2 superconformal field theory is given by a modified version of the Green-Schwarz superstring which describes in $D = 4$ super-space the superstring compactified on a six-dimensional manifold [17]. The left-moving worldsheet fields of this superstring consist of $[x^m, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}, p_\alpha, \bar{p}_{\dot{\alpha}}, \rho]$ plus an N=2 c=9 superconformal field theory which describes the compactification manifold. p_α and $\bar{p}_{\dot{\alpha}}$ are the conjugate momenta to the superspace variables θ^α and $\bar{\theta}^{\dot{\alpha}}$, ρ is a chiral boson, $m = 0$ to 3, and $\alpha, \dot{\alpha} = 1$ to 2. As discussed in [17], this superstring is related to the RNS superstring by a field-redefinition of the worldsheet variables. For this version of the superstring, the four fermionic generators are

$$\begin{aligned} G^+ &= d^\alpha d_\alpha e^\rho + G_C^+, & \tilde{G}^+ &= [e^{\int J}, G^-], \\ G^- &= \bar{d}^{\dot{\alpha}} \bar{d}_{\dot{\alpha}} e^{-\rho} + G_C^-, & \tilde{G}^- &= [e^{-\int J}, G^+], \end{aligned} \quad (8)$$

where $d_\alpha = p_\alpha + i\bar{\theta}^{\dot{\alpha}} \partial x_{\alpha\dot{\alpha}}$, $\bar{d}_{\dot{\alpha}} = \bar{p}_{\dot{\alpha}} + i\theta^\alpha \partial x_{\alpha\dot{\alpha}}$, $J = \partial\rho + J_C$, and $[T_C, G_C^+, G_C^-, J_C]$ are the c=9 N=2 superconformal generators representing the compactification.

In N=4 topological language, the on-shell condition for an N=2 open string vertex operator V is [14]

$$G_0^+ \tilde{G}_0^+ V = 0 \quad (9)$$

where G_0^+ and \tilde{G}_0^+ are the zero modes of two of the four N=4 superconformal generators. This linearized equation of motion is invariant under the linearized gauge invariances $\delta V = G_0^+ \Lambda + \tilde{G}_0^+ \tilde{\Lambda}$. Note that after twisting, the hermitian conjugate of G^+ is \tilde{G}^+ and V is a hermitian string field. For the N=2 superconformal field theory representing the RNS superstring,

the N=2 vertex operator V is related to the usual RNS vertex operator A by $V = \xi_0 A$ (or equivalently, $\eta_0 V = A$). Since (7) implies that $G_0^+ = Q$ and $\tilde{G}_0^+ = \eta_0$, the on-shell condition of (9) is equivalent to the usual RNS on-shell condition $QA = 0$.

So the kinetic action $\int AQA$ is naturally replaced with the action [18]

$$S_{kinetic} = \int V G_0^+ \tilde{G}_0^+ V. \quad (10)$$

Under the U(1) of the N=2 algebra, G^+ and \tilde{G}^+ carry charge +1 and V is neutral, so the open string action violates U(1) charge by +2 as expected for a twisted critical N=2 superconformal field theory. This kinetic action for N=2 superconformal field theories is naturally extended to the non-linear action [18]

$$S_{open} = \int [(g^{-1} G_0^+ g)(g^{-1} \tilde{G}_0^+ g) + \int dt (g^{-1} G_0^+ g)(g^{-1} \tilde{G}_0^+ g)(g^{-1} \partial_t g)] \quad (11)$$

where $g = e^V$ and the open string fields V are glued together using Witten's midpoint prescription. As in the Wess-Zumino-Witten action, this action has the non-linear gauge invariance

$$\delta(e^V) = G_0^+(\Lambda)e^V + e^V \tilde{G}_0^+(\tilde{\Lambda}) \quad (12)$$

which replaces the linearized gauge invariances of (9).

When the N=2 superconformal field theory is chosen to be the open self-dual string, it is easy to show that (11) correctly reproduces the field theory action for self-dual Yang-Mills. When the N=2 superconformal field theory is chosen to be the modified Green-Schwarz open superstring of (8), the action of (11) provides an open superstring field theory action with manifest four-dimensional spacetime supersymmetry which does not suffer from the contact-term divergences of all other open superstring actions. For the uncompactified superstring, one can easily show [18] that the massless contribution to (11) correctly reproduces the action for $D = 10$ super-Yang-Mills written in terms of N=1 $D = 4$ superfields [19]. Note that the gauge invariance of (12) is reminiscent of the gauge transformation of the super-Yang-Mills prepotential in N=1 $D=4$ superspace.

3 New Approach for the Closed Superstring

In this section, some preliminary results are presented for the action for closed superstring field theory. The kinetic action of the open superstring

can be generalized to the following closed string kinetic action:

$$S = \int V(J_0^{++} + \bar{J}_0^{++})(G_0^+ + \bar{G}_0^+)(\tilde{G}_0^+ + \bar{\tilde{G}}_0^+)V \quad (13)$$

where $J^{++} = e^{J^J}$ is one of the SU(2) generators of the N=4 algebra. This action correctly reproduces the kinetic action for self-dual gravity when the N=2 superconformal field theory is chosen to be the closed self-dual string. Furthermore, when the N=2 superconformal field theory is chosen to be the closed RNS superstring of (7) and when V is a (NS,NS) string field satisfying $(G_0^- - \bar{G}_0^-)V = 0$ (which is the N=4 topological analog of $(b_0 - \bar{b}_0)A = 0$), the action of (13) reproduces the (NS,NS) contribution to the kinetic action of (2). This is easy to see since $(G_0^- - \bar{G}_0^-)V = 0$ implies that $V = (G_0^- - \bar{G}_0^-)W$ for some W , so the action of (13) is

$$\begin{aligned} S &= \int V(J_0^{++} + \bar{J}_0^{++})(G_0^+ + \bar{G}_0^+)(\tilde{G}_0^+ + \bar{\tilde{G}}_0^+)(G_0^- - \bar{G}_0^-)W \\ &= \int V(\tilde{G}_0^+ - \bar{\tilde{G}}_0^+)(G_0^+ + \bar{G}_0^+)(\tilde{G}_0^+ + \bar{\tilde{G}}_0^+)W = 2 \int V\eta_0\bar{\eta}_0(Q + \bar{Q})W, \\ &= 2 \int V\eta_0\bar{\eta}_0(Q + \bar{Q})(c_0 - \bar{c}_0)V, \end{aligned}$$

which is the usual RNS kinetic action if one defines $V = \xi_0\bar{\xi}_0 A$ (or equivalently, $A = \eta_0\bar{\eta}_0 V$).

It has not yet been possible to use (13) to derive the Ramond-Ramond contribution to the kinetic action, or to derive the modified Green-Schwarz version of the kinetic action (which would be manifestly spacetime supersymmetric). Note that the Ramond-Ramond contribution to the kinetic action was computed in reference [20], but using a different method [10]. One problem with (13) is that the constraint $(G_0^- - \bar{G}_0^-)V = 0$ is not hermitian since the hermitian conjugate of $(G_0^- - \bar{G}_0^-)$ is $(\tilde{G}_0^- - \bar{\tilde{G}}_0^-)$. Although gauge invariance of (13) does not require that V satisfies $(G_0^- - \bar{G}_0^-)V = 0$, it appears that such a constraint is necessary for introducing interactions.

For generalizing (13) to include interactions, one can adopt either of the two gluing prescriptions. The covariantized light-cone gluing prescription has the advantage that there is a Jacobi-like identity satisfied by switching the order of gluing three strings, i.e. $(A(BC)) = ((BA)C) + ((AC)B)$ where A, B and C are the three strings and $()$ denotes the gluing procedure. Using this gluing prescription and ignoring the hermiticity problem mentioned above, one can define the gauge-invariant non-linear action as

$$S = \int (V(J_0^{++} + \bar{J}_0^{++})(G_0^+ + \bar{G}_0^+)(\tilde{G}_0^+ + \bar{\tilde{G}}_0^+)V + [V, (G_0^+ + \bar{G}_0^+)V](G_0^+ + \bar{G}_0^+)V) \quad (14)$$

where $[A, B]$ is defined by gluing $(\tilde{G}_0^+ - \bar{\tilde{G}}_0^+)A$ with $(\tilde{G}_0^+ - \bar{\tilde{G}}_0^+)B$ (which is anti-symmetric in A and B). If one ignores hermiticity problems, it appears to be also possible to construct a non-linear version of (13) using the Witten-like gluing prescription.

If V satisfies the constraint $(G_0^- - \bar{G}_0^-)V = 0$, then (14) is invariant under

$$\delta V = (G_0^+ + \bar{G}_0^+)\Lambda + (\tilde{G}_0^+ - \bar{\tilde{G}}_0^+)\tilde{\Lambda} + (\tilde{G}_0^+ + \bar{\tilde{G}}_0^+)\Omega + (G_0^- - \bar{G}_0^-)((\tilde{G}_0^+ + \bar{\tilde{G}}_0^+)\Omega (G^+ + \bar{G}^+)V).$$

When the N=2 superconformal field theory is chosen to be the closed self-dual string, (14) reproduces the correct cubic action for self-dual gravity. And when the N=2 superconformal field theory is chosen to be the RNS superstring, (14) reproduces the correct (NS,NS) contribution.

It is easy to show that the covariantized light-cone gluing prescription implies that $[A, [B, C]] + [B, [C, A]] + [C, [A, B]] = 0$, so the interaction term can be interpreted as the large N limit of an open string interaction term where $[,]$ for open strings is the usual commutator coming from the U(N) Chan-Paton factors. This is not so surprising since self-dual gravity can be interpreted as the large N limit of self-dual Yang-Mills, suggesting that the closed self-dual string can be interpreted as the large N limit of the open self-dual string [21]. If such an interpretation also holds for the N=2 superconformal field theory representing the ten-dimensional superstring, it might shed light on the duality relating N=4 super-Yang-Mills with the Type IIB superstring [22].

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